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Heavy flavors in perturbative QCD

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Abstract. I review the techniques used in perturbative QCD to compute open heavy flavor and quarkonium cross sections. I discuss recent theoretical and experimental results, emphasizing the case of bottom production at colliders.

1 Introduction

Heavy flavor production is one of the most extensively studied topics in high-energy particle physics. An impressive amount of data is available, for basically all kinds of colliding particles, which renders it possible to test QCD predictions to some accuracy. Heavy flavor physics will keep on playing a prominent role in the pp program at the LHC and will be of utmost importance in the context of nuclear collisions. The heavy ion community certainly faces daunting problems in understanding heavy flavor signals emerging from a hot and dense environment, problems that can be solved only if the corresponding signals in non-nuclear collisions are well understood and can be used as benchmarks. In order to assess our capability of predicting such benchmark cross sections, in the present paper I shall review the status of QCD calculations and their comparisons with (mainly $p\bar{p}$) data. Given the fact that most of the recent progress has been relevant to the computation of open-b cross section and its comparison to the data, I will emphasize this case in what follows.

2 Open heavy flavors

In open heavy flavor production the final-state observables must be defined using either the variables of the heavy quarks or of the hadrons containing at most one heavy quark and must not contain any reference to quarkonium states. By definition, a quark is heavy when

$$m_Q \gg \Lambda_{\rm QCD}\,. \tag{1}$$

According to this equation, up, down, and strange quarks are definitely not heavy; for the remaining flavors, we have

$$m_t/\Lambda_{\rm QCD} \simeq 800, \qquad \Longrightarrow \qquad \alpha_S(m_t) \simeq 0.1, \tag{2}$$
$$m_b/\Lambda_{\rm QCD} \simeq 15, \qquad \Longrightarrow \qquad \alpha_S(m_b) \simeq 0.21, \tag{3}$$
$$m_c/\Lambda_{\rm QCD} \simeq 4, \qquad \Longrightarrow \qquad \alpha_S(m_c) \simeq 0.33, \tag{4}$$

from which one is entitled to consider the top and the bottom to be heavy, while the case of charm is borderline. I shall treat the charm as heavy in what follows, for the simple technical reason that the condition in (1) allows one to define an open-quark cross section without the need to convolute it with fragmentation functions, and thus puts the charm formally on the same footing as the bottom and the top. On the other hand, since the quark mass typically sets the hard scale of the process, the values of α_S reported in (2)–(4) imply that in the case of charm the perturbative results will be affected by the larger uncertainties and that non-perturbative effects are liable to play a major role.

In perturbative QCD, the production of a pair of heavy quarks $Q\overline{Q}$ in the collision of two hadrons $H_{1,2}$ is written according to the factorization theorem

$$\mathrm{d}\sigma_{H_1H_2\to Q\overline{Q}}(S) = \sum_{ij}\int \mathrm{d}x_1\mathrm{d}x_2 f_i^{(H_1)}(x_1)f_j^{(H_2)}(x_2)\mathrm{d}\hat{\sigma}_{ij\to Q\overline{Q}}(\hat{s}=x_1x_2S). \tag{5}$$

As is well known, the parton density functions (PDFs) $f_i^{(H)}$ cannot be computed in perturbation theory but are universal. On the other hand, the short distance cross sections $\mathrm{d}\hat{\sigma}_{ij\to Q\overline{Q}}$ are process-specific and computable in perturbation theory

$$\mathrm{d}\hat{\sigma} = \sum_{i=2}^{\infty} a_i\alpha_S^i = a_2\alpha_S^2 + a_3\alpha_S^3 + a_4\alpha_S^4 + \ldots \tag{6}$$

The coefficients a_2, a_3, and a_4 explicitly indicated in (6) correspond to the LO, NLO, and NNLO contributions respectively. Although the computation of the LO term is almost trivial, that of the NLO term is not, and its achievement represented a breakthrough in heavy flavor physics at the end of the 80's [1,2]. No exact result beyond NLO

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is currently available, and it does not seem probable that it will for some time. This is worrisome, since the NLO corrections for c and b production are of the same size as the LO contributions (i.e., the K factor is about 2), and the scale dependence for some observables is very large; NNLO terms may thus be numerically sizable and quite relevant to the correct predictions of measured quantities.

Even if NNLO (or higher) contributions were available, one must keep in mind that such *fixed-order* results may still be insufficient to obtain phenomenologically sensible predictions. There are two main issues that need be considered.

(1) Large logs appear in the perturbative coefficients:

$$a_i = \sum_{k=0}^{i-2} a_i^{(i-2-k)} \log^{i-2-k} \mathcal{Q}, \tag{7}$$

where "large" means $\alpha_S \log^2 \mathcal{Q} \gtrsim 1$. $\mathcal{Q}$ *may or may not* depend on the observable. If $\mathcal{Q}$ is large, all terms in the expansion on the RHS of (6) are numerically of the same order, and the convergence of the series is spoiled. The way out is that of keeping only (some of) the leading logs in (7), in such a way that the series of (6) can be summed. This effectively corresponds to rearranging the perturbative expansion; technically, one says that the logs are *resummed*.

(2) The quarks, although heavy, cannot be observed; therefore, the open heavy flavor cross section must be supplemented with the description of the quark-to-hadron transition (called fragmentation), which always involves a quantity, the non-perturbative fragmentation function (NPFF), not computable in perturbation theory. For the single-inclusive p_{T} spectrum, one writes

$$\frac{\mathrm{d}\hat{\sigma}(H_Q)}{\mathrm{d}p_{\mathrm{T}}} = \int \frac{\mathrm{d}z}{z} D^{Q \to H_Q}(z;\epsilon) \frac{\mathrm{d}\hat{\sigma}(Q)}{\mathrm{d}\hat{p}_{\mathrm{T}}}, \tag{8}$$

$$p_{\mathrm{T}} = z\hat{p}_{\mathrm{T}}, \tag{9}$$

where Q is the heavy quark, H_Q is a given heavy-flavored hadron, $\hat{p}_{\mathrm{T}}$ (p_{T}) is the transverse momentum of Q (H_Q), $\mathrm{d}\hat{\sigma}(Q)$ is the cross section for open-Q production, and $D^{Q \to H_Q}$ is the NPFF.

According to (2)–(4), we expect QCD predictions to be fairly reliable in the case of top production. Tevatron results confirm this, and NLO QCD predictions are in good agreement with CDF and D0 data. The inclusion of soft-gluon threshold effects, resummed to NLL accuracy according to the computation of [3], is seen to increase only marginally the NLO result, while substantially reducing the scale uncertainty. Top production appears therefore under perturbative control. More stringent tests will be performed in Run II: the errors on mass and rate will be smaller, and measurements will be performed of more exclusive $t\bar{t}$ observables and of single-top cross section.

QCD corrections are much larger in the case of bottom production, which is also the heaviest quark that hadronizes before decaying. The data for b production at colliders are extremely abundant, especially so for single-inclusive observables that are relatively well measured also

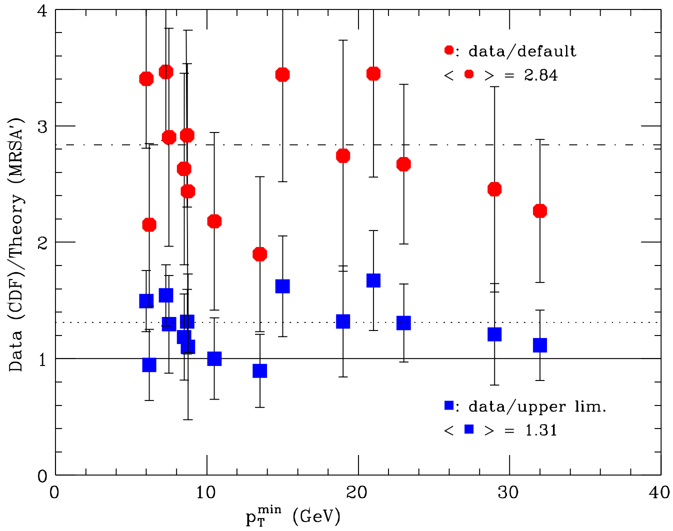

Fig. 1. Ratios of CDF data for b-quark p_{T} spectrum at Run I over NLO QCD predictions

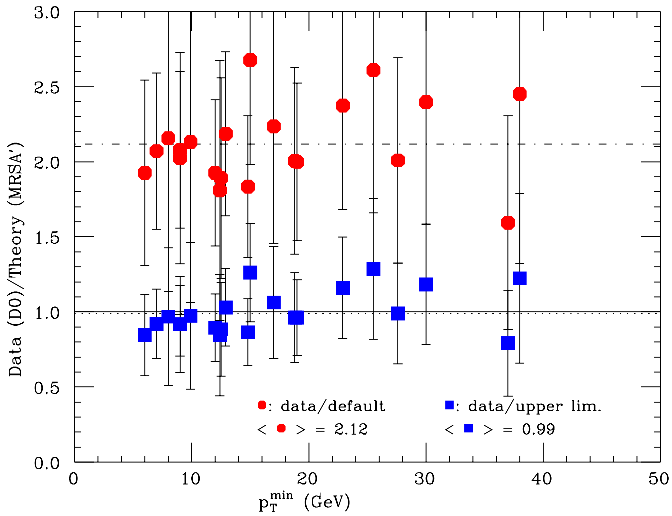

Fig. 2. As in Fig. 1, for D0 data

in absence of vertex detectors. A collection of pre-2000 data, relevant to the single-inclusive p_{T} distribution in the central rapidity region ($|y| < 1$), are presented in Figs. 1 and 2 for CDF and D0 respectively, divided by the pure NLO QCD predictions. When the default values are adopted for the b mass (4.75 GeV), renormalization and factorization scales ($\mu = \sqrt{p_{\mathrm{T}}^2 + m_b^2}$), and Λ_{QCD} (the world average), the ratios are displayed as full circles. NLO QCD, although able to reproduce the shape of the measured distributions, undershoots the data by a factor of 2.84 (for CDF) and 2.12 (for D0) on average. The agreement improves (full squares) if one adopts values for the parameters entering the computation ($m_b = 4.5$ GeV, $\mu = m_T/4$) which are so extreme to be almost "unphysical". The last data available from Run I confirmed the trend of Figs. 1 and 2; as an example, I show in Fig. 3 the measurement presented in [4] of the B^+ p_{T} spec-

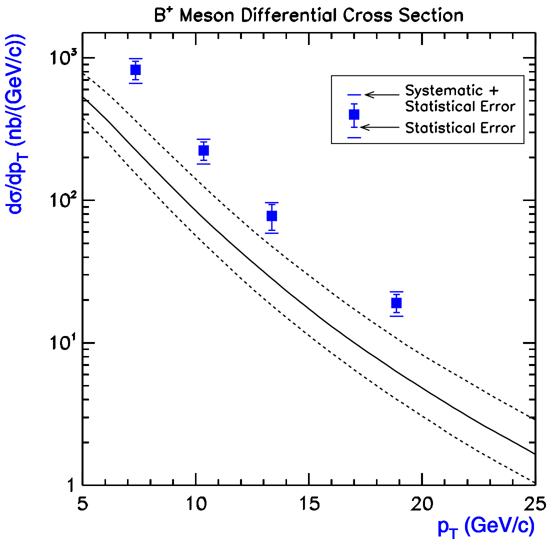

Fig. 3. B^+ CDF data [4] versus NLO QCD predictions, also computed in [4]

trum; CDF find that the average data/theory ratio is $2.9 \pm 0.2 \pm 0.4$.

The disagreement between b production data at the Tevatron and QCD predictions has been one of the most compelling problems in hadronic physics. There are a number of possible explanations for the excess.

(1) New physics: although this remains a viable possibility, it must reconciled with high-precision e^+e^- data. For example, an interesting scenario was proposed [5] which advocates the production of a gluino of 12–16 GeV, with subsequent $\tilde{g} \to \tilde{b}b$ decay with the sbottom of 2–5.5 GeV. Such a scenario, however, appears to be ruled out by LEP data, which exclude at the 95% of CL a sbottom with mass smaller than 7.5 GeV [6].

(2) The QCD predictions used in the comparison with data are not adequate for such a task. This may happen for a variety of reasons:

- Do large logs spoil the convergence of the series?
- Is the fragmentation description not appropriate?
- Do we need to compute higher orders?

(3) The data are (incorrectly) biased by the theoretical predictions (typically obtained from parton shower Monte Carlos) used in the analysis.

The new physics explanation is perhaps the most appealing, but at present it seems premature to adopt it without first reassessing carefully all possible sources of mistakes in the past comparisons between theory and data, and without considering the uncertainties that so-far uncalculated SM contributions can give.

In what follows, I thus focus on a critical analysis of the perturbative predictions used in the past (here in Figs. 1–3) in the comparisons with data. According what discussed above, one possibility is that NLO QCD results must be supplemented by the resummation of some class of large logs in order to give phenomenologically sensible predictions. In general, the logs to be resummed can be divided into two broad classes.

Observable-dependent logarithms: these logs depend on the kinematics of the final state (including cuts); a sample of their arguments is given in the equations below:

$$\mathcal{Q} = \frac{p_T(Q)}{m_Q}, \qquad p_T(Q) \gg m_Q, \tag{10}$$

$$\mathcal{Q} = \frac{p_T(Q\overline{Q})}{m_Q}, \qquad p_T(Q\overline{Q}) \simeq 0, \tag{11}$$

$$\mathcal{Q} = 1 - \frac{\Delta\phi(Q\overline{Q})}{\pi}, \qquad \Delta\phi(Q\overline{Q}) \simeq \pi, \tag{12}$$

of which (10) is relevant to the single-inclusive transverse momentum distributions, whereas (11) and (12) are relevant to $Q\overline{Q}$ correlations. Analytic resummations for the logs of this class are observable dependent and technically fairly involved, which renders the resummed cross sections unavailable except for a few simple cases. Even if the resummed observable can be computed, in general it must be matched to the corresponding fixed-order result for the predictions to be physically meaningful. Fortunately, this is the case for the single-inclusive p_T spectrum: the FONLL formalism [7] allows one to consistently combine (i.e., avoiding over-counting) the NLO result with the cross section in which p_T/m logs are resummed to NLL accuracy. Thus, FONLL can describe both the small-p_T ($p_T \sim m$, where resummed results do not make sense) and the large-p_T ($p_T \gg m$, where NLO results are not reliable) regimes.

The technical complications of the analytic resummations can be avoided by letting a parton shower Monte Carlo (PSMC) perform the resummation numerically. This procedure has the advantage that the logs can always be resummed, no matter how complicated the definition of the observable and the final-state cuts are. The drawback is that the PSMC resummation is formally less accurate in terms of log accuracy than the analytic resummations, although in practice the difference between the two approaches is almost always negligible. On the other hand, PSMC's are based on a LO computation at the level of matrix elements, which is largely insufficient in the case of heavy flavors. In recent years, however, the problem of the consistent inclusion of NLO matrix elements into a PSMC framework has been successfully solved in QCD (MC@NLO [8,9]); phenomenological results obtained with this formalism will be presented later.

Observable-independent logs: these logs do not depend on the kinematics of the final state. Those relevant to heavy flavor production are

$$\mathcal{Q} = 1 - 4m_Q^2/\hat{s}, \qquad \hat{s} \simeq 4m_Q^2, \tag{13}$$

$$\mathcal{Q} = m_Q^2/\hat{s}, \qquad \hat{s} \gg m_Q^2, \tag{14}$$

denoted as threshold and small-x logs respectively. Techniques to resum the former logs are rather well established; their effects are rather marginal, however, in c and b physics, except for b production at HERA-B (their role in top production has been mentioned before). On the other hand, small-x logs are theoretically challenging and intriguing. The standard Altarelli–Parisi evolution equations are replaced by those of CCFM; upon doing so, one

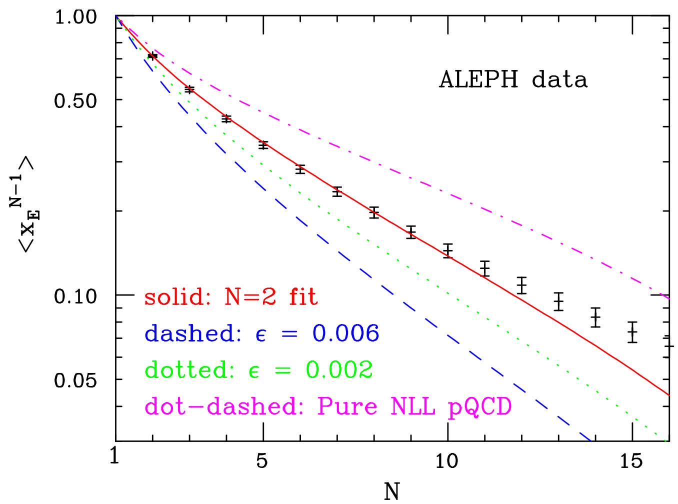

Fig. 4. ALEPH data for the Mellin moments of the energy fraction of the b quarks at LEP, versus theoretical predictions

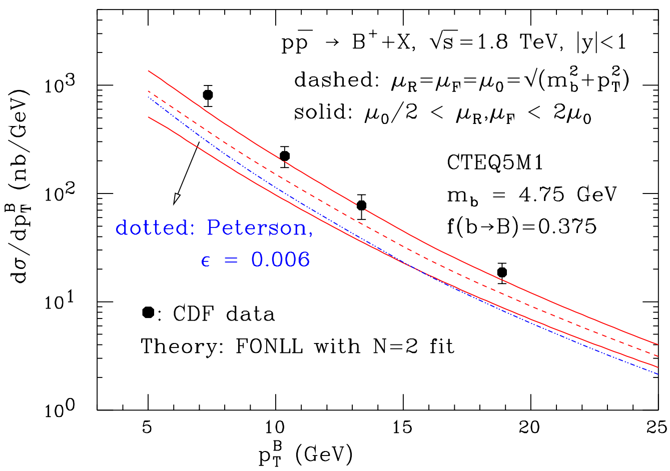

Fig. 5. Same B^+ CDF data as in Fig. 3, compared to QCD predictions based on FONLL with $N = 2$ fits of the fragmentation function

is forced to introduce the so-called unintegrated PDFs, which have a functional dependence on a transverse momentum in addition to those on x and Q^2 of the standard PDFs.

After all the relevant large logs are properly resummed, one needs to understand whether the description of the fragmentation is physically sensible. I remind the reader that the NPFF is not calculable from first principles, and the free parameter(s) it contains (denoted by ϵ in (8)) is (are) fitted to the data after choosing a functional form in z. This fit is typically performed using (8), identifying the LHS with the e^+e^- data. It follows that the value of ϵ is strictly correlated to the short distance cross section $\mathrm{d}\hat{\sigma}(Q)$ used in the fitting procedure and thus is *non-physical*. When (8) is used to predict B-meson cross sections, it is therefore mandatory to make consistent choices for ϵ and $\mathrm{d}\hat{\sigma}(Q)$. This has not been done by CDF in the computation of the theoretical predictions of [4], reproduced here in Fig. 3: for $\mathrm{d}\hat{\sigma}(Q)$, the NLO result of [10] is used, but the value of ϵ adopted (0.006) has been derived in the context of a LO, rather than NLO, computation. On the other hand, if a more appropriate value of ϵ is chosen (∼0.002 [11]), the theoretical prediction increases by a mere 20% [12], still rather far away from the data.

There are, however, a couple of issues that need be taken into account. In the p_T range probed at the Tevatron, large logs of p_T/m may start to show up. Therefore, FONLL results should be used rather than NLO ones. The second observation concerns again the NPFF: $\mathrm{d}\hat{\sigma}(Q)/\mathrm{d}\hat{p}_\mathrm{T}$ has a power-like form, and one writes

$$\frac{\mathrm{d}\hat{\sigma}(Q)}{\mathrm{d}\hat{p}_\mathrm{T}} \simeq \frac{C}{\hat{p}_\mathrm{T}^N} \quad\Longrightarrow\quad \frac{\mathrm{d}\hat{\sigma}(H_Q)}{\mathrm{d}p_\mathrm{T}} = \frac{C}{p_\mathrm{T} B N} D_N^{b\to B}, \quad (15)$$

$$D_N^{b\to B} = \int \mathrm{d}z z^{N-1} D^{b\to B}(z;\epsilon). \quad (16)$$

It turns out that the approximation for the B cross section given in (15) is in excellent agreement with the exact result [13]. Since $N = 3$–5 (at the Tevatron) it follows that, in order to have an accurate prediction for the p_T spectrum in hadroproduction, it is mandatory that the first few Mellin moments computed with $D(z)$ agreed with those measured. In [12], it is pointed out that this is not the case, in spite of the fact that the prediction for the inclusive b cross section in e^+e^- collisions, obtained with *the same* $D(z)$, displays an excellent agreement with the data. There may seem to be a contradiction in this statement: if the shape is reproduced well, why this is not true for the Mellin moments? The reason is that, when fitting $D(z)$, one excludes the region of large z, since it is affected by Sudakov logs and by complex non-perturbative effects which are unlikely to be described by the NPFF. On the other hand, the large-z region is important for the computation of D_N (because of the factor z^{N-1} in the integrand of (16)). Therefore, for the purpose of predicting B-meson spectra at colliders, [12] advocates the procedure of fitting the NPFF directly in the N-space. A fit to the second moment (denoted as $N = 2$ fit henceforth) is found to fit well all the D_N's for N up to 10 (see Fig. 4).

It appears therefore that a better comparison with Tevatron data should be obtained by using FONLL rather than pure NLO predictions, with the fragmentation function obtained with an $N = 2$ fit. The result of such a comparison is shown in Fig. 5, where the data are the same ones are those displayed in Fig. 3: the average data/theory ratio is now $1.7 \pm 0.5 \pm 0.5$ i.e. data are within 1σ from the default theoretical prediction. A factor of 20% of the reduction from the formerly claimed discrepancy of 2.9 to the current 1.7 is due to the use of FONLL in place of NLO results; the remaining 45% to the correct treatment of the NPFF.

These findings suggest to recompute the theoretical predictions upon which Figs. 1 and 2 are based. Unfortunately, this would not help much, since most of the data presented there are relevant to b quarks, rather than to B-mesons; in other words, experimental collaborations deconvoluted the $b \to B$ fragmentation. This has been typi-

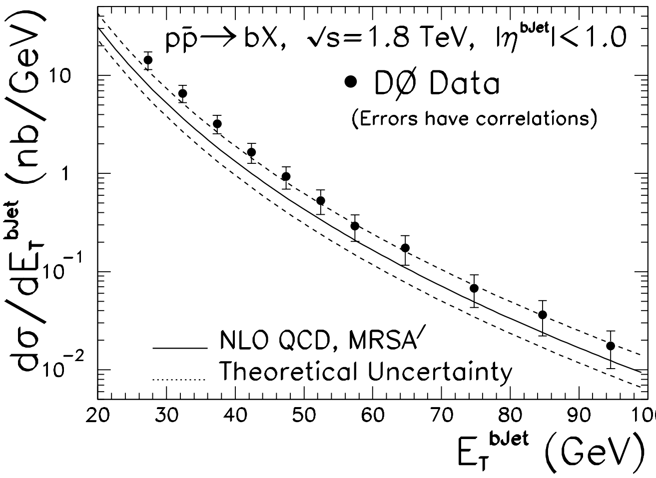

Fig. 6. D0 data for inclusive jets containing at least a b quark versus NLO QCD predictions

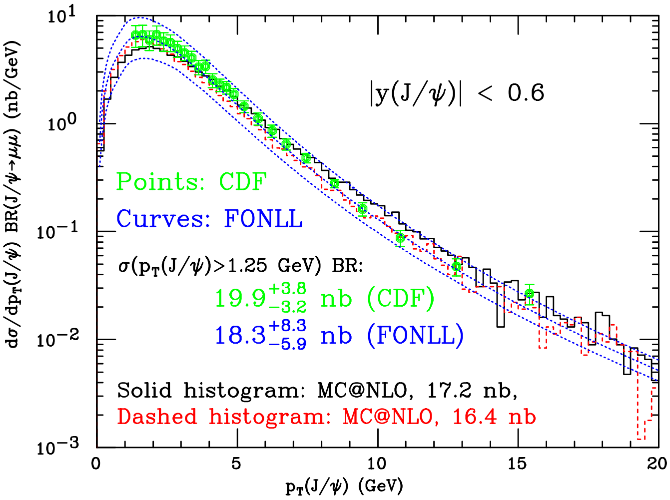

Fig. 7. Comparison of CDF Run II data [16] with MC@NLO [8,9] (solid and dashed histograms, corresponding to different hadronization parameters) and FONLL [7] (dotted curves; the band defines the overall uncertainty) predictions

cally done using PSMC models, and in general it appears rather difficult to recover the data for B-mesons, which is a practical example of the theoretical biases on data mentioned above. On the other hand, there are other ways to understand whether we are on the right track. One interesting possibility consists in considering jets containing b quarks (i.e., any b-hadron species) rather than tagging a specific b-hadron: in this case, the NPFF simply does not enter the cross section, and the theoretical predictions are also less prone to develop large p_T logs, since the p_T of the b is not involved in the definition of the observable. The comparison between NLO predictions for b-jets [14] and D0 measurements [15] is indeed satisfactory: data are consistent with theory in the range $25 < E_T^{b-\mathrm{jet}} < 100\,\mathrm{GeV}$, as shown in Fig. 6.

To fully test the ideas relevant to the NPFF treatment one still needs heavy-flavored meson data. Fortunately, a lot of these are expected to become available in the near future, thanks to the ongoing Run II. The first results on single-inclusive b-hadron p_T spectrum have been presented by CDF [16], and they are particularly interesting in view of the fact that for the first time they probe the region of $p_T \simeq 0$, which is fairly sensitive to the description of the fragmentation. The comparison of the data with FONLL (dotted lines) is presented in Fig. 7, and displays the best-ever agreement between theoretical predictions and b data at colliders. The same pattern of agreement is obtained by using MC@NLO (histograms), which constitutes a very powerful check on the theoretical results: both the resummation and the $b \to B$ transition are performed in vastly different ways in FONLL and MC@NLO. I should stress that the plot presents the p_T spectrum of the J/ψ's emerging from b-flavored hadron decays and thus involves a highly non-trivial combination of short- and long-distance dynamics (see [17] for a more detailed discussion).

Run II also offers the possibility of testing the $N = 2$ fit of the NPFF on c-flavored data. In Fig. 8, taken from [18], the FONLL predictions are compared to CDF data

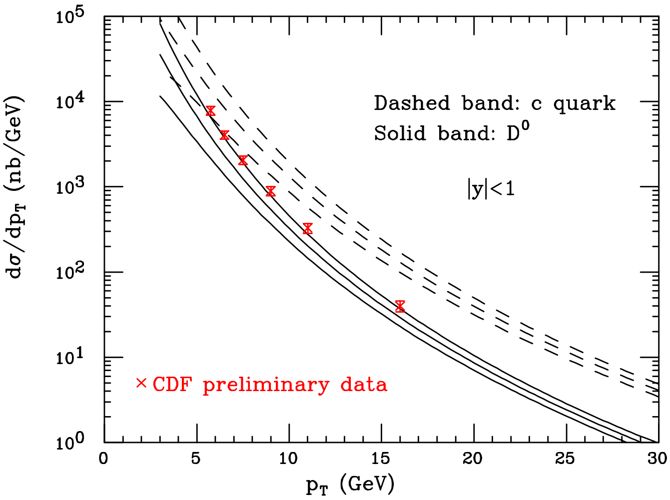

Fig. 8. CDF Run II data for D^0 production versus FONLL predictions

for D^0 production; the pattern of comparison is similar to that of Fig. 5. This test is particularly significant, since the resummation effects are larger than in the case of B production, the mass of the charm quark being smaller than that of the bottom.

Although the tests presented above prove that when comparing modern data sets with up-to-date theoretical predictions heavy flavor production appears to be fairly well predicted by QCD, we must keep in mind that the uncertainties affecting the QCD results are still quite large; thus, the possibility remains that such uncertainties hide physics effects not included in the computations. Within non-BSM physics, one of the most interesting questions is whether b production at the Tevatron is small-x physics. According to [19], small-x resummation would increase the b cross section by a mere 30%. On the other hand, a good description of Tevatron data is obtained by using CASCADE [20], a Monte Carlo code that implements CCFM evolution equations; this approach also gives results in agreement with c-flavored hadron data at the Tevatron (see [21]). It should be noted that, since the (LO) matrix elements convoluted with unintegrated PDFs have off-shell partons, they include part of the contributions which are of NLO in the standard collinear approach. This renders the interpretation of the results more complicated, since it is impossible to tell the *pure* small-x effects apart from higher-order corrections to the matrix elements. It seems therefore necessary to compute the NLO corrections rigorously in the context of the small-x approach in order to achieve firmer conclusions. I should also mention the fact that unintegrated PDFs, exactly like standard PDFs, cannot be computed from first principles and need to be extracted from the data. Such an extraction is at present affected by large uncertainties (especially for the gluon density), which must be systematically reduced in order for small-x computations to be as reliable phenomenologically as those based on collinear factorization.

In conclusion, it appears that at present perturbative QCD is doing an excellent job in predicting open heavy flavor cross sections measured at colliders. The most substantial improvement in recent years occurred in b physics; the long-standing discrepancy between single-inclusive b data at colliders and theoretical predictions has been settled mainly thanks to a better understanding of the non-perturbative phenomena, since the backbone of the computations is still the NLO result of [1,2], which is essential to get anywhere close to the measurements. On the other hand, the careful re-analysis of the computations motivated their improvements, through the matching with resummed results (FONLL) or with Monte Carlo techniques (MC@NLO); the flexibility of the latter guarantees that studies with the same accuracy of those performed so far only for single-inclusive observables can be repeated for basically any type of variable.

The lessons learned now will be very valuable at the LHC, where the uncertainties will be bigger and the differences between c and b physics more pronounced. Even if NNLO results should become available, a very accurate determination of the benchmark cross sections will require the *measurements* of $pp \to Q\overline{Q}$ rates, which can then be extrapolated by means of perturbative QCD computations from $\sqrt{S} = 14\,\mathrm{TeV}$ to $5.5\,\mathrm{TeV}$ with a percent accuracy [22]. As the case of B data at the Tevatron clearly shows, accurate predictions for elementary pp cross sections in nuclear collisions will be obtained only upon using sensible inputs for non-perturbative physics; here, the ideal solution would be that of getting such inputs through the comparison of the results of dedicated pp and pA runs, which would allow one to clearly disentangle new long-distance effects due to the nuclear environment. In the context of the pp program, an exciting possibility is that of using charm data as a probe for small-x studies; if a large p_{T} range could be accessed (say, 0–40 GeV) there could be the possibility of observing directly the onset of the small-x regime, by going from large to small p_{T} values.

3 Quarkonia

The hadroproduction cross section of a $c\bar{c}$ or $b\bar{b}$ quarkonium state H can be written as in (5)

$$\begin{aligned} &\mathrm{d}\sigma_{H_1H_2\to H}(S) \qquad (17)\\ &= \sum_{ij}\int \mathrm{d}x_1\mathrm{d}x_2 f_i^{(H_1)}(x_1) f_j^{(H_2)}(x_2)\mathrm{d}\hat{\sigma}_{ij\to H}(\hat{s}=x_1x_2S). \end{aligned}$$

The forms of the short distance cross sections that appear in (17) depend on the theory or the model adopted to compute them. Among the various approaches, the *only one* that can be mathematically derived from QCD is non-relativistic QCD (NRQCD [23]), an effective theory in which the heavy quarks move at non-relativistic velocities. Within NRQCD, the partonic cross sections are [24]

$$\mathrm{d}\hat{\sigma}_{ij\to H} = \sum_n \mathrm{d}\hat{\sigma}(ij\to Q\overline{Q}[n])\langle\mathcal{O}^H[n]\rangle, \qquad (18)$$

where $\mathrm{d}\hat{\sigma}(Q\overline{Q}[n])$ is the cross section for the production of a $Q\overline{Q}$ pair in a given spin and color state (symbolically, $n = \{c = (1,8); {}^{2S+1}L_J\}$), and the NRQCD matrix elements $\langle\mathcal{O}^H[n]\rangle$ are analogous to PDFs and NPFFs, since they cannot be computed in perturbation theory and are universal; loosely speaking, they are proportional to the probability for the $Q\overline{Q}$ pair in the state n to fragment into the quarkonium state H.

Given the fact that NRQCD is derived from QCD and that pQCD can describe open-Q data, we expect that NRQCD does a good job too. A difficulty, however, is immediately apparent by looking at (18), that features an infinite sum in which each term contains a non-calculable long-distance parameter ($\langle\mathcal{O}^H[n]\rangle$); this implies a complete loss of predictive power. Fortunately, the NRQCD matrix elements obey a (velocity) scaling rule [25]

$$\langle\mathcal{O}^H[n]\rangle \propto v^{f(n,H)}, \qquad v^2 \simeq 0.3, 0.1 \quad \text{for} \quad c\bar{c}, b\bar{b}, \qquad (19)$$

where v is the relative velocity of the $Q\overline{Q}$ pair in the quarkonium state, and f is a function that depends in a complicated manner on the quantum numbers of the states $Q\overline{Q}[n]$ and H (see e.g. [26]). Equation (19), combined with the usual expansion in α_S of the short distance cross sections, implies that the RHS of (18) can be rewritten as a double series

$$\mathrm{d}\hat{\sigma}_{ij\to H} = \sum_{m,k} s_{m,k}\alpha_S^m v^k. \qquad (20)$$

This systematic expansion in α_S and v provides a computational framework similar to that relevant to open-Q production. Still, (20) poses some non-trivial computational problems, given the fact that the double series is slowly "convergent", particularly so for charm (owing to (4) and (19)), and thus one needs to determine a large number of NRQCD matrix elements (some of them can be expressed in terms of others, for example using heavy quark spin symmetry and the vacuum saturation approximation). Furthermore, the same problems that affect the

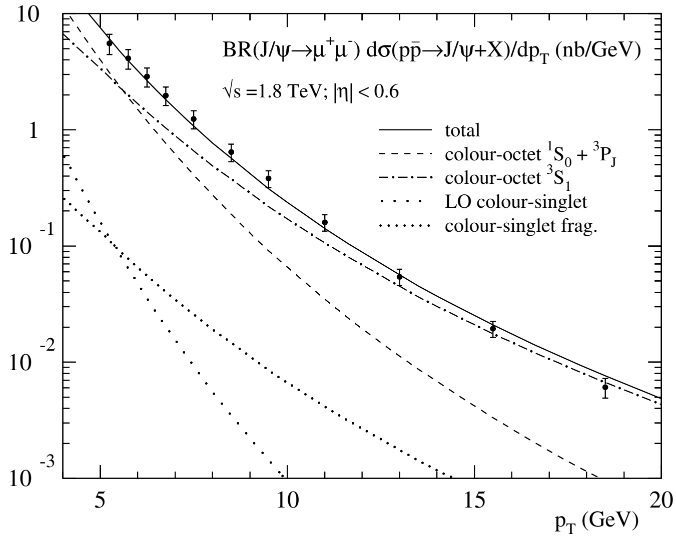

Fig. 9. CDF Run I data [27] for J/ψ production vs NRQCD predictions

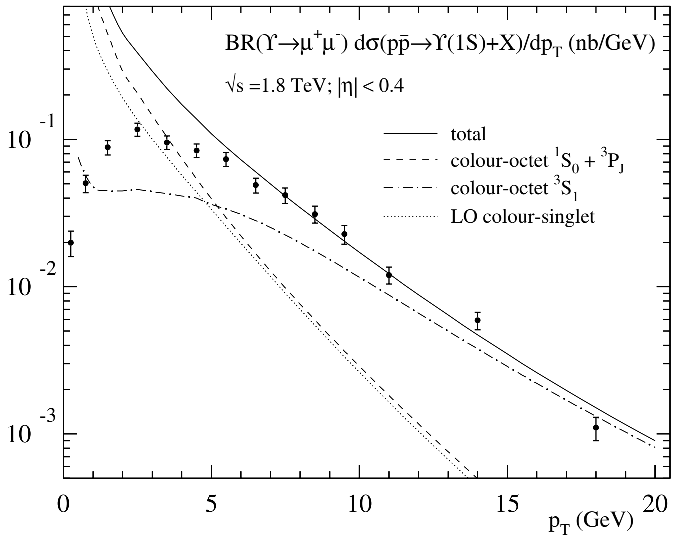

Fig. 10. As in Fig. 9, for Υ production [28]

short distance coefficients of (6) are also relevant to the present case; most notably, the occurrence of large logs can render the expansion (20) useless.

The first stringent experimental tests of NRQCD predictions at colliders have become available only relatively recently, thanks to the use by CDF of microvertices that allow for the precise measurements of the *direct* cross sections (those in which the observed quarkonium state is not obtained through feeddown from more massive bound states). This immediately led to ruling out the so-called color-singlet model (CSM), which can be obtained from (18) by dropping all but the leading color-singlet contribution there. In fact, as can be seen from Fig. 9, no amount of tuning of the NRQCD matrix elements can bring the LO CS contribution in agreement with data (because of the different shape in p_{T}), and CO contributions are therefore necessary. The same pattern can be seen in the case of Υ production (Fig. 10). There, thanks to the fact that the measurements reach the small-p_{T} region not probed in the case of J/ψ production, one can see that the fixed-order short distance cross sections cannot account for the Sudakov suppression at $p_{\mathrm{T}} \to 0$, which renders a resummation necessary.

Although the dominance of CO contributions in quarkonia production at the Tevatron has to be regarded as a highly successful prediction of NRQCD, we must keep in mind that the NRQCD matrix elements are (in part) fitted to the data that the theory is supposed to predict. Such a determination, furthermore, is not only affected by fairly large uncertainties [29] but is also biased by the fact that a definite choice for the PDFs of the colliding hadrons must be made. Within these uncertainties, the NRQCD matrix elements do obey the scaling rules of (19). On the other hand, a more convincing test of NRQCD predictions is the check of the universality of the matrix elements, whose values must be independent of the hard process and therefore, if fitted at the Tevatron, can be used at HERA to predict the quarkonium cross sections there. Unfortunately, the comparison between ep data and NRQCD predictions is largely inconclusive as far as the CO contributions are concerned. The J/ψ energy distribution in photoproduction does not support the growth towards the endpoints of the spectrum which is a consequence of the CO terms; on the other hand, it is known that such regions are strongly affected by large higher-order corrections (in v), and thus a resummation would be necessary in order to draw definite conclusions. On the other hand, the data are in good agreement, for both the energy distribution and the p_{T} spectrum, with the pure CS prediction if NLO effects [30] are taken into account. The situation appears to be more consistent with the findings at the Tevatron in the case of DIS data, although a few glitches remain there as well (especially in the z distribution). The bottom line is that it is hard to draw any conclusion at present; data of larger statistics must be obtained in the present HERA run phase in order to test NRQCD more thoroughly.

It is interesting to observe that a good agreement with the Tevatron data shown in Figs. 9 and 10 is also obtained in the context of the color evaporation model (CEM). When using such a model, the short distance cross sections in (17) are written as follows:

$$\mathrm{d}\hat{\sigma}_{ij\to H}^{(\mathrm{CEM})} = F_H \int_{4m_Q^2}^{4m_M^2} \mathrm{d}m_{Q\overline{Q}}^2 \frac{\mathrm{d}\hat{\sigma}(ij \to Q\overline{Q})}{\mathrm{d}m_{Q\overline{Q}}^2}, \qquad (21)$$

where m_M is the mass of the lowest-lying Q-flavored meson state, and F_H is a universal (long-distance) constant. The color evaporation model can be formally written in the same form as NRQCD, by replacing the original expression for the NRQCD matrix elements

$$\mathcal{O}^H[n] = \chi^* \kappa_n \psi \left(\sum_X |H+X\rangle\langle H+X| \right) \psi^* \kappa_n' \chi, \qquad (22)$$

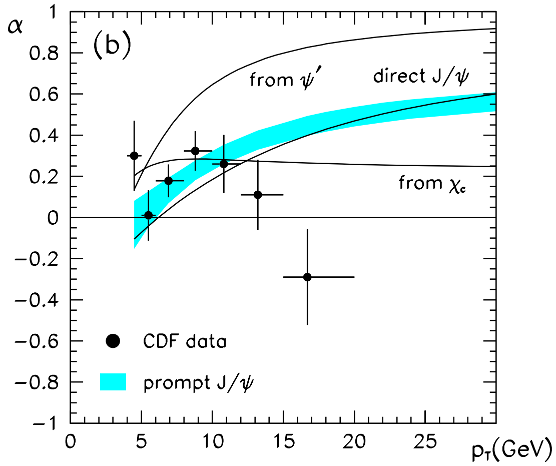

Fig. 11. J/ψ polarization data [31] vs NRQCD predictions

with

$$\mathcal{O}^H[n] = F_H \sum_n \chi^* \kappa_n \psi \sum_X |Q\overline{Q}(m^2_{Q\overline{Q}} < 4m^2_M) + X\rangle \times \langle Q\overline{Q}(m^2_{Q\overline{Q}} < 4m^2_M) + X|\psi^* \kappa'_n \chi. \quad (23)$$

Equation (23) basically entails a change of the scaling rules, $v^{f(n,H)} \to v^{2L}$; this is interesting, since it implies that a re-organization of the double expansion in (20) can also give satisfactory phenomenological results. It should be noted that in general CEM predictions must be supplemented with a $k_{\rm T}$-kick in order to describe the data, which decreases the predictive power of the model. The fact that the only information concerning the quarkonium state is contained in the constant F_H is also troublesome, since for example the ratio $\sigma(\chi_c)/\sigma(J/\psi)$ turns out to be different if measured in hadron–hadron and photon–hadron collisions at fixed-target. Furthermore, a weak $p_{\rm T}$ dependence is found for the J/ψ decay fractions.

A relatively clean measurement which can serve as a powerful test of the underlying theory is that of the polarization of the quarkonium produced in hadronic collisions. A convenient parametrization of the polarization is given in terms of α, introduced as follows:

$$\frac{{\rm d}\sigma_{H\to\mu^+\mu^-}}{{\rm d}\cos\theta} \propto 1 + \alpha\cos^2\theta, \quad (24)$$

with

$$\alpha = \frac{\sigma_{\rm T} - 2\sigma_{\rm L}}{\sigma_{\rm T} + 2\sigma_{\rm L}}, \qquad \theta = \angle(p_{\mu^+}, p_H^{(\rm boost)}). \quad (25)$$

In the context of NRQCD, a simple inspection of the LO diagrams and of their dependence on the transverse momentum allows one to conclude that at large $p_{\rm T}$ the color-octet $^3S_1$ fragmentation contribution is dominant (as can also be seen from Figs. 9 and 10). At large $p_{\rm T}$ the gluon is basically on shell, which implies that it is transversely polarized. Apart from higher-order corrections, such a polarization is transferred to the quarkonium state; thus, the NRQCD prediction is $\alpha = 1$ at asymptotically large $p_{\rm T}$. On the other hand, the CEM results are unpolarized by construction, and therefore $\alpha = 0$ there.

Tevatron data, shown in Fig. 11, strongly disfavor the CEM prediction, and strictly speaking the model is ruled out by this measurement. The attitude can be taken, however, that the CEM is not a fundamental theory, and therefore that one should not expect it to be applicable to any kind of observable, in particular those involving polarizations. The problem is more serious for NRQCD which, being a rigorous consequence of QCD, must be able to predict whatever variable, however complicated. This does not happen in the case of J/ψ polarization, as apparent from Fig. 11, where the predictions of [32] are compared to the data. I must stress that this disagreement poses a much more serious problem to NRQCD than the results obtained at HERA, since the computation of the polarization is regarded as a fairly solid prediction, being dominated by a single contribution. Clearly, higher orders in α_S and v, the feeddown, spin-flip corrections of $\mathcal{O}(v^2)$ can all dilute the polarization; however, unless the coefficient of one of these contributions is anomalously large, all these effects should have a moderate impact on the value of α.

In summary, NRQCD is an elegant and compact formulation which in principle allows the systematic computation of any observable relevant to quarkonium production. Being a direct consequence of QCD, it appears to be fairly solid. On the other hand, problems remain in the comparisons with collider data, the most serious of which is that of the J/ψ polarization. It should be noted that NRQCD computations are not at the same level of accuracy as those used to predict open heavy flavor cross sections; both the v and the α_S expansions must be considered, and the computation of the observables beyond LO (which appears to be a necessity in heavy flavor physics) is extremely complicated. The success of the CEM may suggest that, at least for the case of charm, velocity scaling rules may not be adequate, and more theoretical work is needed in this direction. It appears unlikely that the present situation will substantially change before the start of LHC operations, although measurements with higher precisions will certainly be obtained at Run II. The pp program at the LHC will presumably tell the final word on polarization measurements, with data for Υ available up to large $p_{\rm T}$'s. It is fair to say that at present it is largely unknown the extent to which the ideas of NRQCD, even if fully confirmed by pp data, will survive in nuclear collisions. The use of phenomenological models such as CEM will provide a guideline for benchmark cross sections; as in the case of open heavy quark production, the role of control run will be essential.

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